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THE MARKET FOR INNOVATIONS

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INTRODUCTION

This paper presents a brief survey of the theoretical literature dealing with innovation in a market economy. The survey is organized around the following topics: first, the theory of the production, marketing and use of innovations; second, welfare aspects of innovative activities; and third, factor augmenting bias in innovation. The survey attempts to summarize the present state of knowledge concerning these topics.

It is only fair to state that the economic theory of innovation is still in its infancy, with major problems unsolved. The most important of these arise because of the central role played by uncertainty in innovation. Beyond purely technological uncertainty associated with creating innovations, there are uncertainties arising from responses of rivals and uncertainties due to the speculative nature of the uses to which innovations can be put as well, as other sources of risk to the innovator. Moral hazards and imperfections of the capital market interfere with the use of the market mechanism to eliminate these uncertainties. This is compounded in the case of innovative activity by the problems of inappropriability and indivisibility. It is our view that these latter constitute problems of secondary importance

relative to those arising from uncertainty, but they represent additional sources of difficulty for a market oriented economy, in its allocation of resources to R&D. The theory of innovation is sufficiently advanced so that these basic problem areas can be identified, but on the other hand the theory is of limited value in devising policy measures to solve these problems. For example, it is still a matter of active debate as to whether monopoly power is a help or a hindrance in increasing the rate of technological progress in the economy.

In those areas of economic theory applied to policy analysis that are presently well-formulated, two elements are normally present: (1) a well-defined set of optimality conditions and (2) a description of institutions whose functioning would lead to satisfaction of the optimality conditions. The economic theory of innovations provides neither. Normative statements about innovations are at best based on partial analysis rather than on the appropriate general equilibrium framework, and are often crude. More importantly, the theory gives no indications as to the set of institutions that could lead to optimality. Institutions designed to mandate appropriate behavior in one dimension preclude appropriate behavior in some other dimension. For example, strengthening the patent system may increase the incentive to produce innovations but this entails a reduction in their utilization below an optimal level.

The difficulties involved in creating a viable economic theory of innovation are deep-seated. Economic theory is most informative when dealing with routinized operations; in fact, it is almost the essence of the theory that it reduces complex situations to simplified decision problems that could as well, if not better, be solved by a computer than by the economic actors themselves. After all is said and done, innovation is concerned with the creation of new alternatives; if this process could be routinized, it would no longer qualify as innovation.

What descriptive economic theory in fact provides us with is a model of the process of choosing from among innovations once they are created, and a study of the market processes that either help or hinder this choice process. If product innovations begin to look very much like advertising expenditures, and if process innovations like movements along an isoquant in response to changes in factor prices, this is because the unique character of innovation as an economic phenomenon has been suppressed in order to apply the economic theory of routinized operations to innovation.

Finally, even within the rather narrow range of problem areas concerning innovation where economic theory provides insights, there are major unsolved theoretical issues. The theory of information as a commodity is still very much to be explored, oligopoly theory and the theory of rival responses in situations of confrontation is hung up awaiting a resolution of the prisoner's dilemma paradox, and we know very little concerning the theory of human capital specialized to creative activities. In summary, the problems of innovation cut across so many difficult areas of economic theory that it is not surprising that economists have yet to develop a satisfactory approach even to those problem areas where economists might ultimately provide interesting comments.

The Market for Innovations

The literature dealing with the market for innovations is dominated by two major themes: first, the Schumpeterian hypothesis (see Schumpeter, 1950) that monopoly power leads to a faster rate of technical progress than does classical competition; and, second, the Arrow paradigm (see Arrow, 1962) that views innovations as information, with market problems associated with uncertainty, indivisibilities and inappropriability. These two themes are not unrelated, of course; it is in fact the existence of the market problems identified by Arrow that forms the theoretical justification for the Schumpeterian hypothesis, with its implied conclusion that losses in static efficiency brought about by monopoly power might be more than offset by corresponding gains in dynamic efficiency.

In dealing with the controversies concerning these fundamental themes, it is convenient to isolate certain features of the market for innovations for study. The survey that follows deals with problems associated with the production of innovations; with the marketing of innovations; and with the use of innovations once they are available. A starting point is the notion of innovation itself. Kuznets (1962) defines innovation as "an application of a new way of attaining a useful end," while Arrow (1962) defines invention as "the production of knowledge." Traditionally, the two concepts have been viewed by economists as markedly different. For example, Schumpeter (1950) assumed that invention was exogenous to the economic system, while innovation was endogenous, representing the main activity of the entrepreneur. However Schmookler (1966) has provided convincing evidence that invention is largely demand oriented rather than being determined primarily by the state of knowledge of the sciences. Thus, both invention and innovation fall within the province of the economist. Most of the literature, however, centers its attention on innovation rather than inventions; similarly, the emphasis is on the development aspects of R&D rather than on research.

The Production of Innovations

Turning to the production of innovations, there are a few stylized facts that appear in the literature. Along with others, Nordhaus (1969) points out that the innovating process is typically highly labor intensive; labor is the most flexible factor of production, and capital intensive processes are inappropriate in the face of the uncertainties that abound in the production of innovations. Mansfield (1968a and b) indicates that innovative activities, whether measured in terms of inputs or outputs, are relatively more important (as a percent of sales or of employment) in firms in the medium to large class, rather than in the very largest firms in an industry. The growing importance of corporate financed research, expecially since World War II, is emphasized by Schmookler (1966), suggesting an increase in the amount of "in house" research. On the other hand, Jewkes (1959), Klein (1962) and Salter (1960) note the crucial roles played by small firms and by "outsiders" in important inventions developed in the recent past (nylon, diesel locomotive, jet airplane, Polaroid camera, computers, etc.). Mueller (1962) reinforces this point by noting that a major fraction of Dupont's sales represents products developed by other companies. Thus the issue of scale economies in the production of innovations is still not completely resolved. Perhaps the most convincing theoretical argument for economies of scale in R&D activities is the contention that large firms can reduce the riskiness of R&D by engaging in a number of independent lines of research, avoiding the "all of the eggs in one basket" problem (see Arrow, 1962 and Nordhaus, 1969).

Unfortunately, there is also some evidence that the probability distribution of returns from innovative activity may be such that increased diversification does not reduce risk measured as the variance of returns (see Nordhaus, 1973).

Uncertainties play a central role in all discussions of innovations. In fact, Arrow (1969) characterizes the process of producing innovations as one of "reducing uncertainties." Under the proper conditions, the existence of commodity-option (insurance) markets for the sharing of risks leads to an efficient allocation of resources under uncertainty in a competitive economy (see Arrow 1965, Debreu 1959). However, the presence of "moral hazards" interferes with the working of such markets. Arrow (1962) notes that moral hazards effectively preclude the application of insurance to innovative activies. For example, if an R&D firm could insure against its inability to produce a desired product, the incentive to succeed in innovation would be seriously weakened. *
Arrow concludes that, assuming risk aversion, less than the social optimum amount of resources is allocated to the process of producing innovations.

Marschak (1967) views the production of innovations as a process in which a priori distributions over the parameters characterizing the production map are converted into narrower a posteriori distributions. His model of optimum management of R&D activities emphasizes the sequential nature of decision making in innovation, as well as the gains to be achieved from parallel lines of research in the face of strong uncertainties, following up earlier suggestions by Klein (1962). While both of these aspects of R&D management suggest

* Beyond this, the cost to an insurer of acquiring adequate information concerning the distribution of payoffs from R&D activities no doubt would preclude the economic viability of insurance, even if moral hazards were not present.

possible gains that might be achieved by centralized control of R&D as contrasted with decentralized decision making, in fact Marschak's examples of R&D decision making in military procurement generally indicate losses in efficiency from too rigid management by the Pentagon. Finally, to the extent that centralized direction appears to offer gains in efficiency, this occurs in the development stage of R&D rather than in the research stage. Marschak's conclusions are thus in line with those of empirically oriented economists who agree that the financial requirements of the development phase of R&D lead to a concentration of such work in medium to large firms rather than in small firms.

The Marketing of Innovations

The Arrow paradigm centers on the problems posed for the marketing of innovations by viewing the innovation as disembodied knowledge. As in most of the literature, innovations are assumed to be process innovations, acting to reduce the costs of producing existing goods. In its purest form, the Arrow paradigm is one in which an innovation, once produced, can be applied to the production of unlimited units of other goods with zero marginal cost. This raises the problem of indivisibilities -- while production of the innovation requires the application of certain quantities of factors of production, it can be reproduced without the expenditure of any further resources. The optimum charge for access to the innovation is the marginal cost of reproduction, i.e., zero. The indivisibilities associated with innovations lead to problems similar to those encountered in the classic case of public goods (see Samuelson, 1955). If marginal cost pricing is followed, we obtain the optimum use of the innovation, but there is no incentive to produce the innovation. On the other hand, if property rights to the innovation are vested in the innovator so that licensing or royalty payments can be exacted from users, this restores in part the incentives for production of innovation, but leads to less than optimal use of innovations once produced.

Related to this is the fact that there are difficulties in establishing and protecting property rights in innovations. This is the problem of inappropriability of innovations. Viewed as pure information, innovations suffer from the difficulty that a monopoly position can be maintained only so long as only the innovator possesses the information. Once the innovation is licensed to even one user, that user then acquires the information possessed by the innovator and is in a position to compete with the innovator in the marketing of the innovation. The patent system at best offers only partial protection to the innovator. Schmookler's data indicate a marked fall in patenting following World War II, despite a continuing increase in private R&D activity, as measured by expenditures and the employment of technical personnel. The conventional wisdom holds that patent rights are relatively ineffective as a device for maintaining a monopoly position with respect to an innovation. Imitators, patent infringers and patent suits all act to limit the profits the innovator can capture; the usual estimate is that the innovator probably can obtain only those profits that would accrue in any case simply because of the two or three year lead time the innovator has over rival firms. Thus the absence of protected property rights in innovations acts to lessen the incentives for producing innovations.

In contrast with these views which emphasize the role played by inappropriability in restricting the output of innovations (or of any kind of "pure" information), Hirschleifer (1970) argues that the private value of an innovation bears no necessary relation to its social value. Thus we might have too much in the way of resources devoted to innovative activities rather than too little; in any case, it does not follow from the fact that innovations are "pure" information that inappropriability is inevitable nor that the society produces too little in the way of innovations. The point is that the innovator as possessor of an item of information has available to him such options as investing in the stocks of companies that would benefit from the innovation, margin buying in futures markets or buying up resources specialized to the production of goods to which the innovation could be applied. Having taken a speculative position the innovator then announces his new production process to the world and reaps the rents that would otherwise accrue to firms or to factors. In fact he can license the innovation and in certain circumstances can capture more than the social value of the innovation. Hirschleifer notes that the private value of the knowledge that horse A is the fastest in a race can be far in excess of whatever social value there might be in the knowledge as to swiftness obtained from the running of the race itself.

The Hirschleifer approach leaves the problem of optimal allocation of resources to innovation or other information producing activities in something of a no-man's-land. In order to accurately judge the optimal character of institutions operating in the market for information it is necessary to examine in detail the speculative opportunities afforded the innovator throughout the economic system.

Hirschleifer is well aware that imperfect capital markets and the lack of adequate futures markets limits the extent to which this argument leads to conclusions concerning the allocation of resources to innovations different from those obtained by Arrow. But these imperfections are present in situations unrelated to innovations; for example, in the timing of equipment replacement, with equipment of known characteristics. The problem peculiar to innovations in this regard is the fact that in order to establish the value of the asset he pledges for loans (namely the innovation itself), the innovator is required to divulge information to the banker that can destroy the innovator's monopoly position: in brief, imperfections facing innovators in the capital market are an almost inevitable consequence of the peculiarities of innovations.

It should also be pointed out that speculative activities as envisaged by Hirschleifer involve major risks simply because market prices are influenced by so many other elements beyond the existence or nonexistence of the innovation.

The Use of Innovations

As regards the use of innovations, the literature is concerned primarily with the effect of market structure on the adoption of innovations. There are a number of different approaches that have been taken, and the conclusions derived are quite sensitive to the structures of the models employed. In Arrow's original formulation of the market structure problem (1962), the question posed is the following. Is there a greater incentive to produce innovations for a competitive industry or for the same industry operated as a monopoly? Actually, Arrow contrasts the case of a monopolistic inventor producing a cost saving innovation for a competitive industry with that of a monopolistic firm that produces a cost saving innovation for itself. Incentives are measured by profits that can be captured under ideal circumstances; e.g., patent rights are inviolable. Arrow concludes that incentives are higher in the case of a competitive industry than under monopoly, but even in the competitive case, the innovator cannot capture the entire amount of social gains accruing because of the innovation.

Demsetz (1969) points out that Arrow's conclusion rests on the fact that the output of the competitive industry is larger than it would be under monopoly control; if adjustment is made for the difference in output, then in fact the profits accruing are larger in the monopoly case. Kamien and Schwartz (1970) show that the crucial fact is the elasticity of the demand curve facing the industry;

the more elastic the demand, the larger is the incentive to produce the innovation. It should be noted that Demsetz' criticism is not really relevant to the question of whether dynamic efficiency is greater under monopolistic or competitive control of a given industry, since the relevant aspect of monopoly control is precisely its restriction of output. It should also be pointed out that there is no disagreement among these authors as to the fact that there is no incentive for a competitive firm to produce innovations unless that firm can establish at least partial monopoly rights to the innovation.

Arrow's approach considers only the two polar extremes of pure monopoly and perfect competition. Horowitz (1963) concludes that in an oligopolistic industry, the incentive to adopt an innovation is greatest the fewer are the firms in the industry, and Ruff (1969), using a somewhat more complex model, arrives at much the same conclusion. In their study of the timing of introduction of innovations, Kamien and Schwartz (1974) derive the interesting finding that somewhere between the extremes of perfect competition and pure monopoly is a market structure with the fastest rate of adoption of innovations. In a related paper (1972), they also point out the sensitivity of the decision to introduce an innovation to expectations; the faster is the expected future rate of technical progress, the later are innovations adopted. Finally, Swan, (1970) examines the question as to whether monopolies tend to restrict the range of products offered or tend to keep new products off the market. The conclusion reached is that while monopolies will charge higher prices and will restrict output of whatever they produce, there are incentives for monopolists just as competitors, to market those commodities desired by consumers. On net balance, the theoretical literature on market structure and innovations comes down on the side of the Schumpeterian hypothesis.

Comments

The general impression one gets from the theoretical literature on innovations is wonder that any innovative activity at all takes place. There are large uncertainties involved in the process of producing an innovation; similar uncertainties as to the value of the innovation once it is produced (particularly with respect to rival innovations that might be in the works); and difficulties in maintaining property rights to whatever value attaches to the innovation. In the face of this, we have estimates ranging up to 90% as to the fraction of growth in output that is accounted for historically by innovations. While estimates of rates of return on innovations are relatively high (see Enos, 1962), they do not appear to be high enough to offset the barriers supposedly facing the innovator. This suggests that the paradigm employed in the study of the market for innovations is somewhat misleading. To the extent that the Hirschleifer objections hold, the amphasis on inappropriability is somewhat misdirected, of course. On the other hand, uncertainties in other aspects of innovative activities certainly remain important.

An alternative paradigm is the following. Innovation is still regarded as the production of new information, but instead of this information being disembodied, it is embodied in an expanded stock of human capital. (Denison (1962) emphasizes the human capital aspect of technological progress.) In the human capital paradigm, what is produced in the innovative process is not simply a new product or a new process for producing existing goods, but rather the know-how, embodied in the innovator and his staff, as to doing something new. In this paradigm, the marginal costs of reproducing innovations by other firms are not trivial, involving as they do the training required to instill the requisite knowledge in others. Furthermore, the services of human capital are not subject to the extreme problems of lack of appropriability that are present in the pure information case. There are still indivisibilities -- an innovator's knowledge can be applied by him to any number of units of goods; and there are still uncertainties; but the importance of appropriability as a factor defining the properties of the market for innovation is downgraded. Such a model should incorporate the fact that know-how is in part an organizational phenomenon so that a part of the knowledge acquired adheres to the firm in which the innovation occurs, rather than in specific individuals in the R&D labs.

There is some evidence for the human capital paradigm relative to the pure information paradigm. In particular, it is difficult to explain the lead time accorded in the literature to innovating firms, even when innovations are not patented, except in the context of a human capital approach to innovation. Schmookler's data on the decline of patenting during a period in which more and more complex innovations are being produced similarly lends itself to a human capital interpretation of innovation. Still, it is really an empirical question as to the importance of human capital formation in the innovative process; for certain innovations, the Arrow paradigm might be appropriate, while for others, human capital considerations might dominate.

In the human capital approach to innovation, attention centers on the characteristics of the market for services of skilled and creative innovators. To satisfy incentive compatibility, such markets must offer to prospective employers a share in the quasi-rents that innovators can generate, while offering innovators incentives to sign over such shares. This involves the creation of property rights to the innovator's human capital, accomplished through contracts that assign patent and/or marketing rights to innovations to the employing firm, and that include prohibitions against working for other firms on projects related to those developed while under contract to the employing firm. Thus instruments exist to effect the trasfer of rights to human capital from the innovator to the firm.

The limited evidence cited earlier indicates that medium to large sized firms offer certain economies of scale in R&D activities. The creation of human capital might be furthered by lumpy inputs such as laboratory facilities, and by interactions with other creative individuals. But there is an even more fundamental factor at work in concentrating innovative activity within larger firms, namely the risk avoidance possibilities present in such firms. The creation of innovations is inherently a high risk undertaking; risk averse innovators should be willing to accept salary and profit sharing arrangements paying less on the average than the expected value of the quasi-rents that their innovations can earn, while larger firms, diversified over several innovators and several projects, can spread the risks and operate with a smaller dispersion about expected earnings than can innovators acting as individuals. Thus the concentration of R&D expenditures and output in medium to large sized firms is consistent with the human capital approach to innovations.

It might be noted that the scale economies enjoyed by larger firms that arise from uncertainty in fact reflect simply the presence of those imperfections in the capital market that arise from default risk (see Quirk 1961 and Smith 1972). There are no scale economies of this type for the economic system itself; hence, to the extent that large firms enjoy monopoly power in the market for their outputs, the concentration of R&D activities in such firms tends to lower the rate of innovation as compared to a situation in which capital were available to both small and large R&D operations at the same marginal cost. The only true economies of scale in R&D for the economic system are those associated with lumpy inputs. This suggests the need for public policy directed towards the provision of funds on a co-insurance basis for small independent R&D establishments. This approach seems particularly relevant in light of the thesis propounded by Klein (1974) in his recent book, that emphasizes the importance of competition in R&D as a source of "macrostability" for the economy.

Welfare Economics of Innovation

The significance of much of the work which has been done on the welfare economics of innovation has been vitiated by two related faults: arbitrary choice of a normative standard and reliance on partial equilibrium analysis. At times it appears that the only standard used is the amount of innovation, with more innovation necessarily being better. Leaving aside the difficulties of devising an unambiguous scalar measure of innovation, this standard is absurd because it ignores the cost of producing an innovation. Other standards have been total profits in an industry made up of inter-dependent, innovating firms (Kamien and Schwartz, 1970), and consumers' surplus (Nordhaus, 1969). If one accepts Pareto Optimality as a fundamental normative standard, it is necessary to reject any of these other standards as having welfare significance. It is not true that a project which maximizes consumers' surplus is necessarily Pareto Optimal for all distributions of income, and it is not possible to label a project desirable without considering distribution of income.

Proper evaluation of the welfare implications of various ways in which innovations could be produced must be done in the framework of a general equilibrium analysis. If the analysis does not provide complete answers, it at least makes clear the obstacles which prevent reliance on apparently more convenient approaches. We will outline a potential approach to general equilibrium analysis which seems to underlie some of the more important and reliable analyses.

Even with appropriate normative standards, any diagnosis of market failure in a partial equilibrium context can be at best suggestive, not conclusive. Hirschleifer's (1970) argument that the private value of information may exceed the social value is a case in point. Hirschleifer does not consider the manner in which expectations or beliefs are formed and policed. An analysis of all related markets is necessary to ensure that compensating adjustments elsewhere in the system do not relieve the problem isolated in the partial context.

To formulate a general equilibrium approach we can define an economy as consisting of the following elements: 1) a set of consumers, each of which is endowed with preferences, resources and a title to various fractions of the profits of firms; 2) a set of firms, each of which possesses a set of feasible activities, which may be expressed in terms of a production function; and 3) a commodity space. Innovation appears to have a natural interpretation as a production activity in such a model of an economy. If the output "innovation" is viewed as a commodity, there can be firms which produce innovation as a sole or joint product. A production process with innovation as an input will differ from a production process without such an input.

That is, following a procedure for incorporating externalities (Arrow, 1969) in a general equilibrium model, we can redefine the commodity space to include as many "innovations" as we like. Indeed, this is a natural procedure since when innovations are inappropriable their production does create externalities. An innovation is produced by the use of economic inputs, so that the production of innovation has a natural representation. Such an economy presumably underlies Arrow (1962), and cannot be expected to be well-behaved.

Innovation can be regarded as an addition to a stock (of knowledge of feasible activities, for example) which is an input in production. Whether this stock is private or public must be specified in the model. Since knowledge is in no sense "used up" by being as input, the economy with innovation is essentially inter-temporal. If all futures markets exist, the standard reinterpretation of the commodity space to index goods by time can be used. A similar construction

can be used to incorporate uncertainty, using contingent claims markets. The failure of futures or contingent calims markets to exist for innovations (or other commodities) is a potential source of inefficiency.

If feasible production sets in an economy with commodity space expanded to include innovation are characterized by inappropriability, indivisibility, and uncertainty, we are in a position to diagnose market failure and to arrive at Arrow's (1962) conclusions about "Welfare Economics and the Allocation of Resources to Invention." Both process and product innovation can be included in this interpretation, since preferences over all conceivable products can be assumed with perfect ease. A product innovation than changes the feasible production set for some firm in such a way that in some dimension the set of feasible output levels goes from $\{0\}$ to something larger. If the product can be sold profitably, consumption in that dimension may also become non-zero. Lancaster's (1966) "New Theory of Consumption" may be used to extend preferences from old goods to new goods if the simpler assumption that preferences are defined over an infinite dimensional "potential" commodity space is unpalatable.

This general formulation reveals certain difficulties which lie in wait for any more approximate approach. In the general model a change in the allocation of resources to invention will produce a changed consumption allocation. This allocation produces a welfare allocation, which can be compared to the initial allocation for welfare judgments. There is no ambiguity about the evaluation of technological change. It does not matter if the productivity of some factors falls while that of others rises; it does not matter if some products are cheaper while others become more expensive. The assumption that each consumer has a complete preference ordering implies that any welfare judgment which can be made about an economy with static technology can be made about an economy with changing technology. But when the general equilibrium approach to the welfare economies of invention is abandoned, all the insoluble index number and cost-benefit paradoxes reappear. There become as many alternative measures of technical progress as there are products times factors; in general, as the square of the dimension of the commodity space. Even labor productivity is well-defined only in terms of specific outputs or indices. The index number problems for new products take on the character mentioned by Blaug (1968): because of its physical difference from older products, we cannot put a value on a new product.

Samuelson's analysis of the "Evaluation of Real National Income" (1950) is particularly relevant to the evaluation of technical change. If there is no "forgetting," a technical change is tantamount to a uniform outward shift in a production possibility frontier, and unambiguously would increase welfare if it were free. If resources must be devoted to an innovation, however, a choice among competing p.p.f.'s may be available, and no single p.p.f. may dominate.

The most natural approach might be to attach prices to each output and each factor of production, and to evaluate an innovation in terms of the profits (or quasi-rents) it would create. This is Arrow's (1962) approach, and it is the obvious counterpart to cost-benefit analysis in other areas. It suffers from the same defects -- the informational requirements when price changes are expected to result, the welfare ambiguity of aggregate consumers' surplus measure, and the general impossibility of relating increases in the value of real national income to Pareto optimality. But these difficulties should not obscure two fundamental points: 1) that innovation, or research and development, or technical change, is of economic significance only because of its impact on social welfare through changes in the output of products or the demand for factors, and 2) that any evaluation of the process must be based on information as to each of the input/output ratios of a production activity.

Innovation and the Theory of the Firm

In the previous section we argued that by interpreting a standard general equilibrium model appropriately we can, in principle, make welfare statements about innovation. Such interpretation, while useful for rigorous welfare comparisons, is too general to provide specific theorems regarding the effect of exogeneous changes or policies on levels of inventive output or on welfare through the level of invention. Nor is there much hope of obtaining testable propositions from such a model, and without such propositions it is impossible to verify the underlying theory of innovation. For such purposes a more precise specification of the nature of inventive activity within the innovating firm is needed. The validity of any economic theory of innovation rests on its representation of the nature of the process of innovation.

Little explicit work on the theory of the innovating firm has been done. In this section we will review some of the more interesting models which have been proposed, evaluate them critically, and propose some lines of future research. As an appendix to this report we include a paper reporting some of our own findings along these lines.

One potential source of microeconomic analysis of innovation is the theoretical literature on induced innovation. The original discussion of induced bias in innovation took place in the context of a macroeconomic growth model. As Nordhaus (1969, p. 110) points out, only a few authors ever examined the microeconomic foundations of their theory, and their reflections were rather sketchy (see Samuelson, 1965, Drandakis and Phelps, 1966, Ahmad, 1966). Therefore it should be recognized that we are asking the literature to do things which its authors did not really intend, and that our critical remarks reflect mainly our different interest.

Despite this, is turns out that the constructions used in analyzing some macroeconomic implications of induced bias in innovation are also of use as descriptions of the behavior of a single firm.

Modern analysis of induced innovation appears to take its start from a remark by J. R. Hicks (1932) that a fall in the price of labor relative to capital would induce labor-saving innovations. In the late fifties several authors (Salter, 1960, Fellner, 1959) argued that "the entrepreneur is interested in reducing costs in total, not particular costs such as labor costs . . . ." (Salter, 1960, p. 44), and reached the conclusion that entrepreneurs would pursue specifically labor-saving techniques only if "because of some inherent characteristic of technology, labor-saving knowledge is easier to acquire than capital-saving knowledge" (Salter, ibid.). Charles Kennedy (1964) used the assumption of a trade-off between labor- and capital-saving innovation to give a theoretical justification for the belief that a rising wage-rental ratio induces labor-saving innovation. He proceeded along the lines suggested by Salter, specifying a particular form for characteristics of the innovative process, which can be summarized as follows.

Consider a production function $Y = F(AK, BL)$, where $A(t)$ and $B(t)$ represent factor-augmenting technical progress. (Not all types of technical progress can be represented as factor augmenting (Solow, 1965), but there are no operational procedures for telling which type of progress is occurring (Phelps, 1966). Then let $\dot{A}/A = a$ and $\dot{B}/B = b$. Kennedy postulates a relation $\phi(a, b) = 0$ with the following properties: $da/db<0$ and $d^2a/db^2<0$. That is, the process of knowledge production is such that the better one becomes at economizing on one factor, the worse one is at economizing on the other.

Kennedy's analysis of the implications of such a trade-off examines the consequences of changing factor shares. His approach was criticized by Ahmad (1966) who attempts to construct a micro-economic rationalization of the aggregate relation posited by Kennedy. Ahmad points out that in general the relation should have the form $\phi(a, b, A, B) = 0$, since the rate of factor-saving may depend on how much of that factor is being used, as well as everything else. This formulation, it turns out, generalizes rather easily to a fairly complete theory of the innovating firm. We provide this step in the appendix.

In Ahmad's model the firm chooses which innovation to adopt from an exogeneously given menu. The firm cannot increase the rate of innovation by devoting more inputs to R&D. Thus the classification of exogeneous trands in innovation plays an important role.

For example, Ahmad concludes that "a rise in the price of labor would lead to an innovation which is necessarily labor-saving, if the innovation possibly is technologically unbiased" (p. 349). If on the other hand, the historical innovation possibility is biased it is possible to find that after an increase (fall) in the price of a factor the use of that factor will increase (decrease). It all depends on what is happening exogeneously. Thus we can have a case in which we would have used less of a factor if we had done without technical progress.

This conclusion may appear less paradoxical when we realize that in Ahmad's analysis two different types of technical change accompany substitution. Prices induce substitution along a given isoquant; they also induce the choice of an isoquant from the new set. It would seem reasonable to find both these effects operating in the same direction. But where a new isoquant will be found depends wholly on an exogenous change. The effects cannot be disentangled because there is no way to tell what would have happened if the firm did nothing the activity of R&D is suppressed. That is, time moves the isoquants in a manner entirely unaffected by the decisions of the firm.

Nordhaus (1973) dealt with one part of this problem by analyzing a more general case that that of Ahmad. To avoid terminological confusion, Nordhaus defined "the set of techniques attainable with a given cost C(as) the C-isotech." He analyzes a case in which the location of an isotech may depend on the technique may depend on the technique chosen previously, on obvious generalization. In the same article Nordhaus also pointed out the implicit assumption in much of the literature that the rate of innovation is determined exogeneously. He argued, repeating the analysis of Nordhaus (1969) that unless the production of a new IPF is costly, and represented as an explicit production activity, "then the theory of induced innovation is just a disguised case of growth theory with exogenous technical change."

The assumption that the rate of technical advance is predetermined is quite unnecessary. Nordhaus (1969) has an interesting model of the endogenous determination of the rate of technical change in which there is no choice of direction. He derives some comparative status results: An increase in the price of output, a fall in the rate of interest, or an increase in the "life" of an invention will increase the rate of technical change.

The inclusion of research costs explicitly in a model of technical change, while making for significant endogenous determination of rate and direction, also causes unresolved difficulties. If the rate of technical change cannot be controlled by the firm, then no problem of convexity or increasing returns can arise. If research involves fixed costs, then the failure of convexity is obvious. Less obvious is the fact that unless (a) there are very strong decreasing returns to conventional factors or (b) the cost of obtaining a given innovation rises linearly with the size of the firm, convexity will break down.

All these points have been made by Nordhaus (1967, 1969, 1973). They suggest that in going to a market model we must consider seriously the possibility that in admitting the existence of technical change we force ourselves to abandon competitive models.

It is possible to conjecture what a complete model of the innovating firm built on these foundations would look like. In his original paper on induced bias Kennedy recommended the abandoning of the distinction between factor substitution and technical change. He cited the difficulty of disentangling the two effects empirically and the availability of his own idea of a trade-off between factor-saving innovations. This argument is strengthened by the contention (Griliches, 1962) that no factor substitution is possible without technical change. Hughes (1971) argues that changes in scale also involve research costs and uncertainty.

We define factor substitution as a (costless) shift from one known production process to another. Griliches argues that only a few processes actually in use (defined by their activity vectors) are known with certainty. All others are uncertain in varying degrees. Therefore we do not have a single isoquant, but rather a probability distribution over the output achievable with certain factor combinations. If we add that there may be a cost to factor substitution, (a case analyzed by Pfouts (1964) in a context which did not consider technical change) the distinction between substitution and innovation becomes meaningless. A complete theory of the behavior of the firm under uncertainty would supplant all analysis of technical change. Note that even problems of indivisibility and inappropriability arise, because in a decision problem with uncertainty information has value.

We fear that such a model, while technically interesting and capable of remedying various defects in current theory, would still fail to capture unique features of the innovative process. Its achievement would be the forcing of innovation into the mold of routing operation, and it would not include the real uncertainties and creativity associated with innovation.

Several authors have also examined the effects of uncertainty on the manner in which innovation proceeds. The analysis raises some interesting issues, but it is fundamentally inconclusive as to policy.

According to Arrow (1962) and others invention is characterized by non-insurable uncertainty. It is further asserted that in the absence of other market failure this implies that if "inventors" are risk averse, the resulting amount of invention will be less than the amount needed to support a Pareto optimum. The theoretical justification of the conclusion about the welfare economics of invention is not given in detail but the conjecture is very likely.

Let us consider the justification of the statement that risk aversion leads to reduction in inventive activity. The expected value of innovation must exceed its cost for a firm to innovate if it is risk averse. If there is a schedule of investment opportunities of decreasing yield, the risk averse firm will naturally find some investments unacceptable which are acceptable to the risk neutral firm. That is fine if all investments have the same riskiness. But innovation may differ in uncertainty from other investments. If an analysis of portfolio choice is used, the conclusion as to the impact of changes in risk-taking becomes ambiguous. In this case it is necessary to place innovation in a list of options with varying riskiness.

Depending on where invention falls, a reduction in risk-taking could increase or reduce invention. If all other risks are insurable, or subject to reduction through contingent claims or futures markets, invention might fall at the riskier end of the spectrum. On the other hand, if rivalry exists, not innovating may

be the riskier strategy. In that case a decrease in risk-taking might increase the amount of innovation. Thus we have no theoretical reason for assuming that decreasing risk aversion in general will increase innovation. In addition, tax policy affects innovation differently from other investment decisions only through its effect on risk-taking. Otherwise a corporate income tax will reduce the incentive to innovate by reducing net returns from the innovation, so that we could increase both innovation and investment by decreasing net taxes. But if it is believed that taxes are desirable despite their effect on invention, a much more complex argument is needed to justify reducing taxes to stimulate innovation.

The analysis thus far should establish that the preferences of the firm with regard to uncertain prospects, and its beliefs about the relative riskiness of investing in innovation, determine the amount of innovation the firm will pursue. Several authors in the field of public finance have analyzed the relation between the corporate income tax and the willingness of the corporation to take risks.

We distinguish between two types of income tax: those which do not alter the first-order conditions for present value maximization (neutral taxes) and those which do. Consider neutral taxes first.

Ths most commonly discussed neutral tax is one in which complete loss offset is allowed, investment expenditures can be charged off as current expenses, and refunds are paid instantly on negative revenue (E. C. Brown, 1948).

The provision of complete loss offset and the allowance for expensing of investment means that the level of investment which maximizes the present value of the firm is unchanged by a tax on profits (Musgrave, 1959).

The tax laws now discriminate in this regard against small firms. In a large firm expensing works as a loss offset since there are other items of net income against which the loss can be written off. If a firm is so small (relative to the risks it incurs) that it has negative income in a year, additional provisions for refunding the (negative) tax on a loss are needed. Since this does not exist, expensing would discourage small firms from risky endeavors while encouraging large firms. The tax write-off implies that any expenditure is reduced at the rate of taxation, while returns are reduced in the same proportion. The marginal investment, whose present value is zero, is unchanged. Inframarginal investments will be reduced in value, but will still have positive present value, and will continue to be adopted.

Musgrave (1959) argues that a "neutral" corporate income tax is not a usable policy instrument because it gives the government no revenue. He assumes constant returns and competitive conditions, so that every investment opportunity earns precisely the market rate of interest. Then by refinding X per cent of the firm's investment and receiving X per cent of its returns the Treasury obtains no more money than it could obtain by selling bonds.

This result is incorrect if it is assumed that there is a schedule of investments, or decreasing returns to investment. Suppose the present value of an investment declines as more money is put into it. Then even with loss offset the Treasury now shares in the investor's infra-marginal returns, and takes in a positive net revenue, since its only option is to lend at the market interest rate, which is less than the yield on all but the marginal investment.

A simple example will establish this point. Suppose the firm invests at time 0, and gets output at time 1 which it sells for p per unit. The market rate of interest is r, and the production function is $y = f(X)$, where $f^1(X) > 0$, $f^{11}(X) < 0$. The firm will maximize the present value of profits

$$\frac{pf(X)}{1+r} - \omega X.$$

The first order condition is

$$\frac{p}{1+r} f^1(X) - \omega = 0.$$

Now introduce a tax t, with instant depreciation and loss offset. Then the present value is

$$\frac{(1-t)p\,f(X)}{1+r} - (1-t)\omega x.$$

The first-order condition is unchanged by this tax -- whatever maximizes the present value will also maximize (1-t) times the present value.

Decreasing returns imply, by Euler's Theorem, that for fixed X, $f^1(X)\,X < f(X)$. At the optimum (which is independent of t),

$$f^1(X) = \frac{\omega(1+r)}{p}.$$

Therefore $f^1(X)\,X = \frac{\omega(1+r)\,X}{r}$,

and

$$f(X) > f^1(X)\,X \text{ imply}$$

$$f(X) > \frac{\omega}{p}(1+r)\,X.$$

Therefore

$$pf(X) > (1+r)\omega X \text{ and}$$

$$tpf(X) > t\,(1+r)\,\omega X.$$

But $t(1+r)\omega X$ is what the government would get at time 1 if it invested the money which it refunds to the firm as a loss offset because of instant depreciation, and $tpf(X)$ is what it actually gets at time 1 in tax revenue. There is a net revenue to the Treasury in this case, which increases with diminishing returns.

Musgrave (1959) and Domar and Musgrave (1944) argue that with complete loss offset and adequate depreciation on increase in the tax rate reduces both expected losses and gains in the same proportion, and that the expected value of earnings is reduced in the same percentage for all assets. Risk and income are reduced proportionately. They assume decreasing marginal utility of income and increasing marginal utility of risk (and separability). With these assumptions and full loss offset the investor will increase the risk he bears in order to increase his income when the tax rate increases. As we argued before, this may or may not increase innovation, depending on how relatively risky innovation is seen to be.

In the case of the non-neutral tax, we have both on income and a substitution effect in regard to risk. The result is that an increase in a tax without loss offset may increase risk-taking or may: decrease risk-taking (Musgrave, 320). If the marginal utility of income of the investor is constant -- a case which we might expect to be true of a corporation with perfect access to funds increasing a non-neutral tax reduces risk-taking. Under the same conditions a neutral tax affects risk-taking not at all. It does appear that under any conditions, making a tax closer to neutrality will not decrease risk-taking, and may increase it.

The neutral tax may affect innovation through another route. Musgrave shows that although the decision to invest does not affect the amount of investment if the firm is risk neutral, it does reduce the size of entrepreneurial returns. If innovation is an entrepreneurial function, increasing even a perfect tax would reduce innovation.

Although these examples suggest that there may be a relation between corporate income taxes and innovation, its direction can only be determined by empirical research. How risky innovation

is, and how the decision to innovate is affected by reduction in entrepreneurial returns must be known. Every effect on innovation is accompanied by another of apposite sign. But we do dnow that if the firm does not face capital constraints, and if there is no entrepreneurial function, changes in income tax will have no special effect on innovation.

Thus we find that the theory of the relation between tax policy and investment suggests that any change in tax policy can either increase or decrease innovation. Quantitative estimates of the size of effects are needed to assess the overall impact.

Moreover, the sensitivity of results to the nature of the firm's preferences, and the difficulty of determining these preferences for a joint stock corporation (Smith, 1974), suggests that we are unlikely to be able to predict the effects of tax policy for some time. We are in no better state with respect to other mechanisms for risk spreading.

Conclusions

This survey has highlighted four major areas which should have high priority in funding of theoretical research. We realize that the R&D Assessment Program has assigned a relatively low priority to theoretical work in general, but we argue that this policy is very mistaken. The gaps in our ability to construct sensible theory applicable to the innovative process are so large that economists have little to offer the policy making process beyond some limited empirical observations. This limitation will not be removed until the theoretical foundations are repaired.

The four areas to which we assign high priority are 1) the development of adequate welfare criteria for judging the social and economic desirability of policies affecting R&D; 2) the investigation of the effect of uncertainty on decision-making in R&D, and in particular the specification of risk-spreading institutions, appropriate to the innovative process, which minimize adverse incentive effects; 3) the exploration of the implications for appropriability of inventions of the hypothesis that technical knowledge is embodied in individuals or research teams, and 4) the construction of detailed models, incorporating uncertainty in a fundamental way, of how the nature and direction of innovative activity responds to market forces. In this context, research is needed to explore the policy relevance of Hirschleifer's view of the market structure for information as a commodity.

These four areas have been defined in the body of our report, where we have surveyed the present state of understanding of these problems and indicated in more detail how future research might proceed.

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APPENDIX

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FACTOR BIAS AND INNOVATIONS: A MICROECONOMIC APPROACH*

by

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FACTOR BIAS AND INNOVATIONS: A MICROECONOMIC APPROACH

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1. This paper is concerned with a classic question in the theory of innovations, namely, the question "does an increase in the wage rate lead towards an increase in the production and use of labor-augmenting innovations?" Section 2 of this paper contains a brief survey of the literature dealing with this question. Most of the existing literature takes a macroeconomic approach to the problem of factor augmentation bias and in addition directs its attention only at the use of innovations, with the menu of innovations available to the society being taken as given exogenously. Our approach in this paper is microeconomic, in order to deal explicitly with the issue of allocation of resources to the production of innovations and with the responsiveness of that production to the price signals provided by the factor markets. Because of aggregation problems the conclusions derived in this paper do not necessarily carry over in a direct fashion to macro models of the economy, but at least certain issues are raised by those conclusions that are relevant to the study of innovation in a macro context.

Briefly, the model that we employ is that of a profit-maximizing firm that produces a final output and also engages in "in house" R & D activities that result in the production of labor- and capital-augmenting innovations specialized to the final product production process. The firm is assumed to be a monopolist in the market for its final product.[1]

We assume that the firm's production activities can be summarized in terms of well-behaved neoclassical production functions. The notion of an "innovations possibilities set" is introduced at the level of the firm, and we show that the set is convex, with strict convexity occurring only if there is decreasing returns to scale in the production of innovations. These results are the microcounterparts of the Kennedy postulate of convexity of the aggregate innovations possibilities set.

The profit-maximizing choices of the firm are derived from a control-theory formulation of the firm's activities. The wage-rental ratio is assumed to be fixed over time, and we attempt to analyze the consequences for labor- and capital-augmenting innovations of a shift in that wage-rental ratio.[2]

By considering a relatively extensive model of the innovating firm, we can examine several complicating features of innovation. The first is the dependence of current innovation possibilities on past innovative activity. The second is the influence of changes in factor prices on the choice of innovative inputs, since the same factors as are augmented may themselves be used to produce innovations. Each of these considerations can produce paradoxical results as to the influence of prices on the direction of technical change. We establish first those few properties of technical change which do not depend on specific assumptions on the form of the dependence of current innovations possibilities on past innovative activity. Then by examining simple cases we show how the complications described make it impossible to state in general how changing factor prices will affect innovation.

Specific results are obtained for two special cases: first, the case in which the percentage rate of increase in augmentation is independent of the levels of such augmentation; and second, the case where the output of factor-augmenting innovations is independent of the levels of augmentation. These two cases exhibit quite different "comparative dynamic" properties, and illustrate the dependence of the conclusions reached concerning the

responsiveness of outputs of innovations, on the assumptions made concerning the characteristics of the production processes employed by the firm.

2. At least since J. R. Hicks' conjecture in the Theory of Wages that increases in the wage rate call forth labor-saving innovations, it has been recognized that changing factor prices may affect innovation. The conjecture is, as we shall show, not obviously true. Nor has it been unchallenged. Fellner (1962) and Salter (1960) have argued precisely the opposite, that although an anticipated change in prices might bias inventive activity, there is no reason to expect a difference in innovation under conditions of continuing high wages than under continuing high profits. The arguments by both authors are based on the idea that the firm does not care what kind of costs are reduced; it simply wants to reduce total costs as quickly as possible.

Two separate points appear at issue. One relates to exogenous trends in innovation or innovation possibilities, and disagreements arise from differing priors on the direction of exogenous trends.[3] The other point, which we address in this paper, is whether and how, in a world with no exogenous trend, innovations will respond to prices.

Despite the important role which technical progress has played in models of economic growth, the problem of determining how the bias in technical change will respond to changes in factor prices has never been the subject of a complete formal analysis. Analysis of induced technical change has concentrated mainly on finding conditions under which the economy will have a long-run balanced growth path consistent with a limited set of "stylized facts." In the major recent treatments of induced innovation [Samuelson (1965), Conlisk (1969), Nordhaus (1969), Drandakis and Phelps (1966)], it is assumed that the quantity of labor to be employed is determined exogenously, and the quantity of capital is determined by saving and investment. Equilibrium and optimal growth paths are found by appropriate optimizations using labor and capital supplies to define constraints. This approach is fundamentally macroeconomic.

Although these models focus on the question of how to choose from among an exogenously given set of innovation possibilities, they arrive at results which are special cases of the model in which innovations are literally produced. A brief survey of these results will set the stage for the models of this paper.

Samuelson (1965) examines a formal model of innovation using Kennedy's (1964) idea of an innovation possibility frontier [IPF]. We revise his notation to be consistent with ours. Assume a production function $F(\cdot,\cdot)$ which is homogeneous of degree one. If technical progress is factor augmenting we can define the variables of the production function to be $A(t)K$, $B(t)L$, so that $Y = F(A(t)K, B(t)L)$. The various derivatives will be represented as follows:

$$\frac{\partial F}{\partial[A(t)K]} = F_1 \qquad \frac{\partial F}{\partial[B(t)L]} = F_2$$

$$\frac{dA}{dt} = \dot{A} \qquad \frac{\dot{A}}{A} = a \qquad \frac{dB}{dt} = \dot{B} \qquad \frac{\dot{B}}{B} = b.$$

Some identities will be used frequently:

$$\frac{\partial F}{\partial K} = AF_1 \qquad \frac{\partial F}{\partial L} = BF_2.$$

Since F is homogeneous of degree one we can define

$$v = \frac{A(t)K}{B(t)L} \text{ and } f(v) = F\left(\frac{A(t)K}{B(t)L}, 1\right) = \frac{1}{B(t)L} F(A(t)K, B(t)L)$$

where $f'(v) = F_1$

$f - vf'(v) = F_2.$

We can call v the "augmented capital-labor ratio."

The share of capital in output, α_K, can be defined in terms of F_1 or of f'. Let r = price of K, w = price of L. Then

$$\alpha_K = \frac{rK}{Y} = \frac{\partial F}{\partial K}\frac{K}{F} = \frac{AKF_1}{F} = \frac{vf'}{f}$$

Similarly

$$\alpha_L = \frac{wL}{Y} = \frac{\partial F}{\partial L}\frac{L}{F} = \frac{BLF_2}{F} = \frac{f - vf'}{f}$$

$$\frac{\alpha_K}{\alpha_L} = \frac{vf'}{f - vf'}.$$

Following Samuelson we define a cost function

$$C\left(\frac{r}{A(t)}, \frac{w}{B(t)}\right)$$

where $C = \min_{K,L} rK + wL$ subject to

$$F(A(t)K, B(t)L) = 1.$$

Samuelson exploits certain "duality" relations between C and F. Define

$$C_1 = \frac{\partial C}{\partial\left(\frac{r}{A(t)}\right)} \qquad C_2 = \frac{\partial C}{\partial\left(\frac{w}{B(t)}\right)}, \text{ so that } \frac{\partial C}{\partial r} = \frac{C_1}{A(t)} \qquad \frac{\partial C}{\partial w} = \frac{C_2}{B(t)}.$$

Samuelson states that

$$\frac{\partial C}{\partial r} = \frac{K}{F}, \qquad \frac{\partial C}{\partial w} = \frac{L}{F},$$

and that

$$\frac{rK}{CF} = \alpha_K = \frac{K}{F}\frac{\partial F}{\partial K},$$

$$\frac{wL}{CF} = \alpha_L = \frac{L}{F}\frac{\partial F}{\partial L}.$$

Finally, Samuelson assumes an exogenous innovation possibility frontier, written

$$b = f(a)$$

where $f'(a) < 0, \qquad f''(a) < 0.$

Samuelson assumes that firms act to minimize the instantaneous rate of reduction in unit cost. The rate is obtained by evaluating $\frac{\partial C}{\partial t}/C$.

$$\frac{\partial C}{\partial t} = C_1 \frac{d}{dt}\left(\frac{r}{A(t)}\right) + C_2 \frac{d}{dt}\left(\frac{w}{B(t)}\right)$$

$$= -\left[C_1 r\frac{\dot{A}}{A^2} + C_2 w\frac{\dot{B}}{B^2}\right] + C_1\frac{\dot{r}}{A} + C_2\frac{\dot{w}}{B}$$

$$= -\left[\frac{\partial C}{\partial r} r\frac{\dot{A}}{A} + \frac{\partial C}{\partial w} w\frac{\dot{B}}{B}\right] + \frac{\partial C}{\partial r}\dot{r} + \frac{\partial C}{\partial w}\dot{w}$$

$$= \left[\frac{rK}{F}\cdot\frac{\dot{A}}{A} + \frac{wL}{F}\cdot\frac{\dot{B}}{B}\right] + \frac{K}{F}\dot{r} + \frac{L}{F}\dot{w}.$$

$$-\frac{\dot{C}}{C} = \frac{rK}{CF}a + \frac{wL}{CF}b - \left[\frac{K}{CF}\dot{r} + \frac{L}{CF}\dot{w}\right]$$

$$= \alpha_K a + \alpha_L b - \left[\frac{K\dot{r} + L\dot{w}}{CF}\right]$$

The firm chooses a and b to maximize this expression subject to $b = f(a)$. Substituting and differentiating we have

$$\frac{d}{da}\left[\alpha_K a + \alpha_L f(a)\right]$$

$$= \alpha_K + \alpha_L f'(a) = 0$$

$$f'(a) = -\frac{\alpha_K}{\alpha_L}.$$

Under the assumed conditions $f'(a) < 0$, $f''(a) < 0$ we can solve for the inverse function

$$g\left(\frac{\alpha_K}{\alpha_L}\right) = f'^{[-1]}\left(\frac{\alpha_K}{\alpha_L}\right),$$

where $g' = \frac{1}{f''} < 0$. By successive eliminations we can obtain an expression

$$-a = h\left(\frac{r}{A}/\frac{w}{B}\right).$$

By using the first order conditions we obtain α_K and α_L in terms of AK and BL; further substitution enables us to express AK and BL in terms of the ratio $\frac{r}{A}/\frac{w}{B}$. Thus

$$h' = g'\cdot\frac{d\left(\frac{\alpha_K}{\alpha_L}\right)}{d\left(\frac{AK}{BL}\right)}\cdot\frac{d\left(\frac{AK}{BL}\right)}{d\left(\frac{r}{A}/\frac{w}{B}\right)}$$

We can determine the sign of each derivative. By assumption

$$\text{sgn } g' = -.$$

By convexity

$$\text{sgn}\,\frac{d\left(\frac{AK}{BL}\right)}{d\left(\frac{r}{A}/\frac{w}{B}\right)} = -.$$

The sign of $d\left(\frac{\alpha_K}{\alpha_L}\right)/d\left(\frac{AK}{BL}\right)$ depends on the elasticity of substitution.

$$\frac{d\left(\frac{\alpha_1}{\alpha_2}\right)}{d\left(\frac{AK}{BL}\right)} = \frac{d}{dv}\left(\frac{vf'}{f - vf'}\right)$$

$$= \frac{(f - vf')(vf'' + f') - vf'(f' - vf'' - f')}{(f - vf')^2}$$

$$= \frac{vff'' + f'(f - vf')}{(f - vf')^2}$$

$$= \frac{f'vff''}{f'(f - vf')^2} + \frac{f'(f - vf')}{(f - vf')^2}.$$

Since $\sigma = -\frac{f'(f - vf')}{vff''}$,

$$\frac{d\left(\frac{\alpha_K}{\alpha_L}\right)}{d\left(\frac{AK}{BL}\right)} = \frac{f'}{-\sigma} + \frac{f'}{f - vf'}$$

$$= \frac{f' - \sigma f'}{-\sigma(f - vf')}$$

$$= \frac{\sigma - 1}{\sigma} \; \frac{f'}{f - vf'}.$$

Since $f' > 0$ and $f - vf' > 0$ by assumption,

$$\frac{d\left(\frac{\alpha_K}{\alpha_L}\right)}{d\left(\frac{AK}{BL}\right)} \gtreqless 0 \longleftrightarrow \sigma \gtreqless 1.$$

Therefore $h' \gtreqless 0 \longleftrightarrow \sigma \gtreqless 1$.

Drandakis and Phelps (1966) assume that the firm maximizes the instantaneous proportionate rate of growth in output with fixed inputs. Writing $\dot{Y} = F(K, L, t)$, they define

$$R = \frac{F_t}{F},$$

which is to be maximized. Bias they define as

$$D = \frac{\partial^2 F}{\partial K \partial t} - \frac{\partial^2 F}{\partial L \partial t} = M_K - M_L.$$

Assuming factor augmentation, they obtain an expression

$$R = \frac{\partial F[A(t)K, B(t)L]}{\partial t} \Big/ F$$

$$= \frac{F_1 K\dot{A}}{F} + \frac{F_2 L\dot{B}}{F} = \frac{\dot{A}}{A}\frac{\frac{\partial F}{\partial K}K}{F} + \frac{\dot{B}}{B}\frac{\frac{\partial F}{\partial L}L}{F}$$

$$= \alpha_K \frac{\dot{A}}{A} + \alpha_L \frac{\dot{B}}{B}$$

$$= \alpha_K a + \alpha_L b,$$

which is the same maximand as Samuelson's.

In an Appendix, Drandakis and Phelps derive the following expression for bias

$$D = \frac{1 - \sigma}{\sigma}(b - a).$$

Thus whether or not an increase in the augmentation coefficient for a factor results in a reduction in the use of that factor (in natural units) depends on the elasticity of substitution. For $\sigma < 1$ the expected results hold.

3. We next examine the properties of a model of in house R & D conducted by a firm that produces a final product that incorporates the innovations produced by the R & D labs. We are particularly interested in the way in which profit-maximizing behavior leads to an allocation of resources between the production of capital-augmenting and labor-augmenting innovations in response to the market wage-rental situation.

The model is as simple as possible. The firm is assumed to be a monopolist in the market for its final output Y, produced using labor L_1 and capital K_1 hired in competitive factor markets. Labor and capital used in the production of the final product are augmented by innovations produced in the R & D operations of the firm. Thus AK_1 and BL_1 represent the effective amounts of capital and labor employed in producing the final output.

R & D activities are summarized in two production activities, one producing capital-augmenting innovations, $\dot{A}$, with the other producing labor-augmenting innovations, $\dot{B}$. Innovations are assumed to be specialized to labor and capital employed in producing the firm's final output, so that the labor and capital used in the R & D departments are not augmented by the innovations produced. L_2 and K_2 represent labor and capital employed to produce $\dot{A}$ while L_3 and K_3 denote labor and capital used in the production of $\dot{B}$.

Before turning to the conditions characterizing a profit-maximizing time path for the firm, we first explore the properties of the "innovations possibilities set" for the firm. The "in-house" innovations possibilities set is defined as those combinations of $\dot{A}$ and $\dot{B}$ that can be obtained for a given cost in terms of capital and labor, assuming that A and B are fixed.

To obtain the outer boundary of this set, we solve the following problem.

$$\text{Max } \dot{A} = \phi(K_2, L_2, A, B)$$

subject to (1) $\dot{B} = \overline{C}$

(where $\dot{B} = \Psi(K_3, L_3, A, B)$);

(2) $w(L_2 + L_3) + r(K_2 + K_3) = \overline{M}$

ϕ and Ψ are assumed to be homogeneous of some positive degrees less than or equal to 1, and ϕ and Ψ are quasi-concave.

Let $H = \phi + \lambda_1(\Psi - \overline{C}) + \lambda_2(\overline{M} - w(L_2 + L_3) - r(K_2 + K_3))$.

At a constrained maximum we have

(i) $\phi_K - r\lambda_2 = 0$

(ii) $\phi_L - w\lambda_2 = 0$

(iii) $\lambda_1\Psi_K - r\lambda_2 = 0$

(iv) $\lambda_1\Psi_L - w\lambda_2 = 0$

(v) $\Psi - \overline{C} = 0$

(vi) $\overline{M} - w(L_2 + L_3) - r(K_2 + K_3) = 0$

$$\text{Let} \quad A^* = \begin{bmatrix} \phi_{KK} & \phi_{KL} & 0 & 0 & 0 & -r \\ \phi_{LK} & \phi_{LL} & 0 & 0 & 0 & -w \\ 0 & 0 & \Psi_{KK} & \Psi_{KL} & \Psi_K & -r \\ 0 & 0 & \Psi_{LK} & \Psi_{LL} & \Psi_L & -w \\ 0 & 0 & \Psi_K & \Psi_L & 0 & 0 \\ -r & -w & -r & -w & 0 & 0 \end{bmatrix}$$

then at a regular constrained maximum, A^* has the property that $|A^*| > 0$.

The slope of the boundary of the innovations possibilities set is given by

$$\frac{d\dot{A}}{d\dot{B}} = \frac{\phi_K dK_2 + \phi_L dL_2}{\Psi_K dK_3 + \Psi_L dL_3}$$

From (2), $w dL_2 + r dK_2 = -(w dL_3 + r dK_3)$

while from (i) - (iv),

$$\frac{d\dot{A}}{d\dot{B}} = \frac{r\lambda_2 dK_2 + w\lambda_2 dL_2}{r\left(\frac{\lambda_2}{\lambda_1}\right)dK_3 + w\left(\frac{\lambda_2}{\lambda_1}\right)dL_3} = -\lambda_1$$

It thus follows that the innovations possibilities set is convex if $\frac{d\lambda_1}{dB} \geq 0$.

Differentiating the first order conditions with respect to $\dot{B}$ we obtain the system

$$\begin{bmatrix} \phi_{KK} & \phi_{KL} & 0 & 0 & 0 & -r \\ \phi_{LK} & \phi_{LL} & 0 & 0 & 0 & -w \\ 0 & 0 & \Psi_{KK} & \Psi_{KL} & \Psi_{K} & -r \\ 0 & 0 & \Psi_{LK} & \Psi_{LL} & \Psi_{L} & -w \\ 0 & 0 & \Psi_{K} & \Psi_{L} & 0 & 0 \\ -r & -w & -r & -w & 0 & 0 \end{bmatrix} \begin{bmatrix} dK_2 \\ dL_2 \\ dK_3 \\ dL_3 \\ d\lambda_1 \\ d\lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ d\dot{B} \\ 0 \end{bmatrix}$$

Hence $\frac{d\lambda_1}{d\dot{B}} = \frac{A^*_{55}}{|A^*|}$ where A^*_{55} is the cofactor formed by deleting the fifth row and column from A^*.

By block multiplication we have

$$A^*_{55} = \begin{vmatrix} \phi_{KK} & \phi_{KL} \\ \phi_{LK} & \phi_{LL} \end{vmatrix} \cdot \begin{vmatrix} \Psi_{KK} & \Psi_{KL} & -r \\ \Psi_{LK} & \Psi_{LL} & -w \\ -r & -w & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \Psi_{KK} & \Psi_{KL} \\ \Psi_{LK} & \Psi_{LL} \end{vmatrix} \cdot \begin{vmatrix} \phi_{KK} & \phi_{KL} & -r \\ \phi_{LK} & \phi_{LL} & -w \\ -r & -w & 0 \end{vmatrix}$$

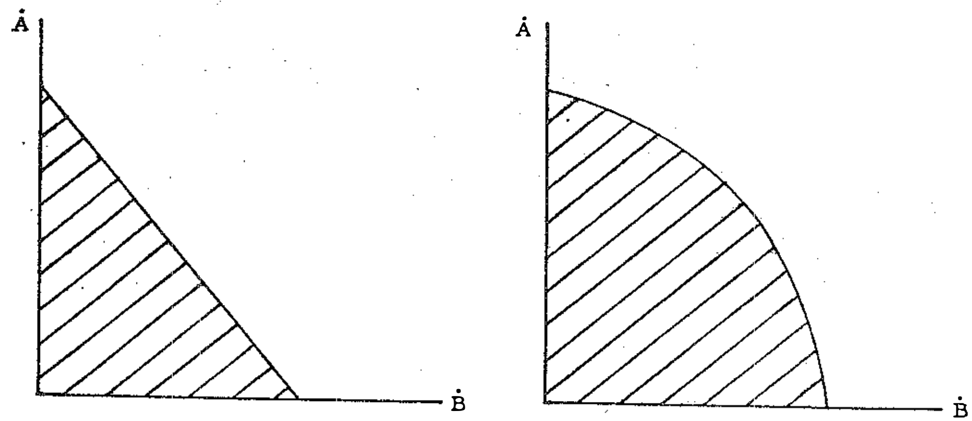
If ϕ and Ψ are homogeneous of degree one, then $A^*_{55} = 0$ and $\frac{d\lambda_1}{d\dot{B}} = 0$. If ϕ and Ψ are homogeneous of positive degree less than one,

note that $\begin{vmatrix} \Psi_{KK} & \Psi_{KL} & -r \\ \Psi_{LK} & \Psi_{LL} & -w \\ -r & -w & 0 \end{vmatrix} = \left(\frac{\lambda_1}{\lambda_2}\right)^2 \begin{vmatrix} \Psi_{KK} & \Psi_{KL} & \Psi_{K} \\ \Psi_{LK} & \Psi_{LL} & \Psi_{L} \\ \Psi_{K} & \Psi_{L} & 0 \end{vmatrix}$

while $\begin{vmatrix} \phi_{KK} & \phi_{KL} & -r \\ \phi_{LK} & \phi_{LL} & -w \\ -r & -w & 0 \end{vmatrix} = \left(\frac{1}{\lambda_2}\right)^2 \begin{vmatrix} \phi_{KK} & \phi_{KL} & \phi_{K} \\ \phi_{LK} & \phi_{LL} & \phi_{L} \\ \phi_{K} & \phi_{L} & 0 \end{vmatrix}$

Under quasi-concavity and positive homogeneity of degree less than one, both of these expressions are positive.

Hence we conclude that when ϕ and Ψ exhibit constant returns, the IP set is convex with a straight line outer boundary. Under decreasing returns to scale, homogeneity and quasi-concavity, the IP set is convex with the outer boundary a strictly concave function. The cases are shown below.



Constant returns to ϕ and Ψ — Decreasing returns to ϕ and Ψ

The In-House I. P. Set

(Note that because A and B are assumed fixed, precisely the same diagrams appear if we replace $\dot{A}$ by $\dot{A}/A$ and $\dot{B}$ by $\dot{B}/B$).

4. Turning to the implications of profit maximization for the allocation of resources within the firm, the firm's problem may be formulated as follows.

$$\max \int_0^{\infty} [p(Y)F(AK_1, BL_1) - wL - rK]e^{-\delta t}dt$$

Subject to $\dot{A} = \phi(A, B, K_2 L_2)$

$\dot{B} = \Psi(A, B, K_3 L_3)$

$A(0) = A_0 \qquad B(0) = B_0$

where $L = L_1 + L_2 + L_3$

$K = K_1 + K_2 + K_3, \qquad Y = F(AK_1, BL_1)$

Let $H = [pF - \omega L - rK]e^{-\delta t} + \lambda_1\phi + \lambda_2\Psi$

First order conditions are given by:[4]

(1) $[(MR)F_1 A - r]e^{-\delta t} = 0$

(2) $[(MR)F_2 B - w]e^{-\delta t} = 0$

(3) $-re^{-\delta t} + \lambda_1\phi_K = 0$

(4) $-we^{-\delta t} + \lambda_1\phi_L = 0$

(5) $-re^{-\delta t} + \lambda_2\Psi_K = 0$

(6) $-we^{-\delta t} + \lambda_2\Psi_L = 0$

(7) $\dot{\lambda}_1 = -(MR)F_1K_1e^{-\delta t} - \lambda_1\phi_A - \lambda_2\Psi_A$

(8) $\dot{\lambda}_2 = -(MR)F_2L_1e^{-\delta t} - \lambda_1\phi_B - \lambda_2\Psi_B$

(9) $\dot{A} = \phi$

(10) $\dot{B} - \Psi$

with transversality conditions $\lim_{t\to\infty} \lambda_1 = 0, \lim_{t\to\infty} \lambda_2 = 0;$

where $MR = p + \frac{dp}{dY}, F_1 = \frac{\partial F}{\partial(AK_1)}, F_2 = \frac{\partial F}{\partial(BL_1)}.$[5]

We will work with the special case in which p(Y) is assumed to satisfy $\lim_{Y\to 0} p(Y) = +\infty$, $\lim_{Y\to+\infty} p(Y) = 0$, $p(Y) \geq 0$ for all $Y \geq 0$ with $\frac{dp}{dY} < 0$ for all $Y \geq 0$. Further, F, ϕ, and Ψ are assumed to be "well behaved" neoclassical production functions. In particular F is homogeneous of degree one in AK_1, BL_1, while ϕ and Ψ are homogeneous of degree one in K_2, L_2, and K_3, L_3 respectively.

Thus $F(AK_1, BL_1) = BL_1F(v_1, 1) \equiv BL_1f(v_1)$ where $v_1 = \frac{AK_1}{BL_1}$. Further, $\lim_{v_1\to 0} f(v_1) = +\infty$, $\lim_{v_1\to+\infty} f(v_1) = 0$, $f(v_1) \geq 0$ for $v_1 \geq 0$ and $f'(v_1) > 0$, $f''(v_1) < 0$ for $v_1 \geq 0$.

Similarly, let $\phi(A, B, K_2, L_2) = \alpha(A)G(K_2, L_2) = \alpha(A)L_2g(v_2)$ while $\Psi(A, B, K_3, L_3) = \gamma(B)H(K_3, L_3) = \gamma(B)L_3h(v_3)$ where $v_2 = \frac{K_2}{L_2}$, $v_3 = \frac{K_3}{L_3}$. g and h are assumed to obey the same neoclassical properties as f.[6]

Finally, let $\mu_1 = \lambda_1e^{\delta t}$, $\mu_2 = \lambda_2e^{\delta t}$. Thus the first order conditions may be rewritten as follows.

(1') $[(MR)Af' - r]e^{-\delta t} = 0$

(2') $[(MR)B(f - v_1f') - w]e^{-\delta t} = 0$

(3') $-r + \mu_1g'\alpha = 0$

(4') $-w + \mu_1(g - v_2g')\alpha = 0$

(5') $-r + \mu_2h'\gamma = 0$

(6') $-w + \mu_2(h - v_3h')\gamma = 0$

(7') $\dot{\mu}_1 = \delta\mu_1 - (MR)f'K_1 - \mu_1\alpha' L_2 g$

(8') $\dot{\mu}_2 = \delta\mu_2 - (MR)(f - v_1f')L_1 - \mu_2\gamma' L_3 h$

(9') $\dot{A} = \alpha(A)L_2g(v_2)$

(10') $\dot{B} = \gamma(B)L_3h(v_3)$

Let $\omega = \frac{w}{r}$. Then (1') - (6') can be used to establish that

$$\omega = \frac{B}{A}\left[\frac{f - v_1 f'}{f'}\right] = \frac{g - v_2 g'}{g'} = \frac{h - v_3 h'}{h'}$$

We are particularly concerned with the case where ω is constant over time. For that case we have

$$\dot{\omega} = 0 = \frac{-gg''}{(g')^2}\dot{v}_2 = \frac{-hh''}{(h')^2}\dot{v}_3$$

Under the neoclassical conditions, v_2 and v_3 are uniquely determined by ω, and are constant over time when ω is constant.

Further, given ω, A and B, v_1 is uniquely determined, while $\dot{\omega} = 0$ implies that

$$\frac{\dot{v}_1}{v_1} = \sigma_f \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)$$

where σ_f is the elasticity of substitution between augmented capital and augmented labor in the production of the firm's final product.

Let $q_1 = \frac{K_1}{L_1}$ so that $v_1 = \frac{A}{B} q_1$. We then obtain

$$(*) \quad \frac{\dot{q}_1}{q_1} = (\sigma_f - 1)\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right).$$

This result corresponds to that obtained by Samuelson (1965). It asserts the following.

(i) If $\sigma_f = 1$ so that F is Cobb-Douglas, the capital-labor ratio in producing the firm's final product is independent of the relative rates of innovation so far as capital and labor augmentation is concerned;

(ii) If $\sigma_f > 1$, then the capital-labor ratio in producing the final product increases if the rate of increase in capital-augmenting innovations exceeds the rate of increase in labor-augmenting innovations, and conversely;

(iii) If $\sigma_f < 1$, the capital-labor ratio in producing the final product falls if the rate of increase in capital-augmenting innovations exceeds the ratio of increase in labor-augmenting innovations, and conversely.

Thus far we have only exploited the cost-minimizing properties of the model. The properties that follow from profit maximization can be derived as follows.

Note that conditions (1') - (10') are all identities in t. Differentiating (3') with respect to t gives

$$-\dot{r} + \mu_1 g'\alpha'\dot{A} + \mu_1 \alpha g''\dot{v}_2 + \alpha g'\dot{\mu}_1 = 0$$

For $\dot{\omega} = 0$, $\dot{v}_2 = 0$ and $\dot{\mu}_1 = \frac{\dot{r}}{\alpha g'} - \mu_1 \frac{\alpha'\dot{A}}{\alpha}$.

From (7'), $\dot{\mu}_1 = \delta\mu_1 - (MR)f'K_1 - \mu_1 \alpha' L_2 g$.

Since $\dot{A} = \alpha L_2 g$, we have

$$(MR)f'K_1 = \delta\mu_1 + \frac{\dot{r}}{\alpha g'} = \frac{\delta r + \dot{r}}{\alpha g'}.$$

Similarly, using (5') and (8') we have

$$(MR)(f - v_1 f')L_1 = \frac{\delta r + \dot{r}}{\gamma h'}.$$

It follows that

$$\left(\frac{f'}{f - v_1 f'}\right) q_1 = \frac{\gamma h'}{\alpha g'},$$

which implies $q_1 = \frac{A}{B}\omega\left(\frac{\gamma h'}{\alpha g'}\right)$, in turn leading to

$$(**) \quad \frac{\dot{q}_1}{q_1} = \left(\frac{\dot{A}}{B} - \frac{\dot{B}}{B}\right) + \frac{\gamma'\dot{B}}{\gamma} - \frac{\alpha'\dot{A}}{\alpha}.$$

This condition might be contrasted with that derived earlier, namely

$$(*)\qquad \frac{\dot{q}_1}{q_1} = (\sigma_f - 1)\left(\frac{\dot{A}}{A} - \frac{\dot{A}}{B}\right)$$

Both (*) and (**) must hold simultaneously along a profit-maximizing time path. There are of course a multitude of cases that might be of interest in the study of factor bias and innovation. Here we look only at those cases in which a "quasi-steady state" is achieved in the sense that the capital-labor ratios in all three activities (F, ϕ, Ψ) are constant over time, i.e., cases in which $\dot{q}_1 = 0$ is an identity in t.[7]

For such cases, the analysis of "comparative dynamics" is particularly simple, especially in attempting to determine the impact on the system of a change in the wage-rental ratio ω.

Case 1. $\dot{A}/A$ and $\dot{B}/B$ independent of A and B.

Recently, Ahmad (1966) and Nordhaus (1973) have emphasized the critical importance of the level of technological progress on the rate of advance of such progress.[8] Their comments are best understood by considering the case in which the rate of progress is independent of the level already achieved. In terms of the model developed here, this is the case where $\alpha(A) = C_1A$, $\gamma(B) = C_2B$ for some constants C_1, C_2. We take C_1, C_2 to be unity. From (**) $\dot{q}_1 = 0$ for all t with $q_1 = \omega\left(\frac{h'}{g'}\right)$. If $\sigma_f \neq 1$, then $\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0$ from (*) and hence $v_1 = \frac{A}{B}q_1$ is also constant over time at the level $v_1 = \frac{A}{B}\cdot\omega\left(\frac{h'}{g'}\right)$. Since $\frac{\dot{A}}{A} = L_2g(v_2)$, $\frac{\dot{B}}{B} = L_3h(v_3)$, we have $L_2g(v_2) = L_3h(v_3)$.

Consider a change in ω and its effects on the system.

From $q_1 = \omega\frac{h'}{g'}$, we have

$$\frac{dq_1}{d\omega} = \omega\left(\frac{g'h''\frac{dv_3}{d\omega} - h'g''\frac{dv_2}{d\omega}}{(g')^2}\right) + \frac{h'}{g'}$$

$$\text{But}\qquad \frac{dv_3}{d\omega} = \frac{-(h')^2}{hh''} > 0,\quad \frac{dv_2}{d\omega} = \frac{-(g')^2}{gg''} > 0,$$

$$\text{hence}\qquad \left(\frac{dq_1}{d\omega} = \omega\frac{-(h')^2}{g'h} + \frac{h'}{g}\right) + \frac{h'}{g'}$$

$$= \omega\frac{h'}{g'}\left(\frac{-h'}{h} + \frac{g'}{g} + \frac{1}{\omega}\right)$$

$$\therefore\qquad \frac{1}{q_1}\frac{dq_1}{d\omega} = \left[\frac{-1}{\omega + v_3} + \frac{1}{\omega + v_2} + \frac{1}{\omega}\right] > 0.$$

Thus, when $\dot{A}/A$ and $\dot{B}/B$ are independent of A and B, an increase in the wage-rental ratio increases the capital-labor ratios in both final goods production and in the R & D operations of the firm.

Further, $\omega = \frac{B}{A}\left(\frac{f - v_1f'}{f'}\right)$ so that

$$d\omega = \left(\frac{f - v_1f'}{f'}\right)d(B/A) - B/A\left(\frac{ff''}{(f')^2}\right)dv_1$$

$$\text{and}\qquad \frac{d\omega}{\omega} = \frac{A}{B}d(B/A) + \frac{1}{\sigma_f}\frac{dv_1}{v_1},$$

$$\text{where}\qquad dv_1 = \frac{A}{B}dq_1 + q_1d(A/B)$$

$$\text{with}\qquad \frac{dv_1}{v_1} = \frac{dq_1}{q_1} + \frac{B}{A}d(A/B)$$

$$= \frac{dq_1}{q_1} - \frac{A}{B}d(B/A).$$

$$\therefore\qquad \frac{d\omega}{\omega} = \frac{A}{B}d(B/A)\left[\frac{\sigma_f - 1}{\sigma_f}\right] + \frac{1}{\sigma_f}\frac{dq_1}{q_1}.$$

For $\sigma_f \neq 1$,

$$\frac{A}{B}\frac{d(B/A)}{d\omega} = \left[\frac{\sigma_f}{\sigma_f - 1}\right]\left\{\frac{1}{\omega} - \frac{1}{\sigma_f q_1}\frac{dq_1}{d\omega}\right\}$$

$$= \left[\frac{\sigma_f}{\sigma_f - 1}\right]\left\{\frac{1}{\omega} - \frac{1}{\sigma_f}\left(\frac{1}{\omega} + \frac{1}{\omega + v_2} - \frac{1}{\omega + v_3}\right)\right\}$$

and

$$\frac{\omega}{(B/A)}\frac{d(B/A)}{d\omega} = \left[\frac{\sigma_f}{\sigma_f - 1}\right]\left\{1 - \frac{1}{\sigma_f}\left(1 + \frac{\omega}{\omega + v_2} - \frac{\omega}{\omega + v_3}\right)\right\}$$

$$= \frac{1}{\sigma_f - 1}\left\{\left(\sigma_f - 1\right) + \frac{\omega(v_3 - v_2)}{(\omega + v_2)(\omega + v_3)}\right\}$$

$$= 1 + \frac{1}{(\sigma_f - 1)}\frac{\omega(v_3 - v_2)}{(\omega + v_2)(\omega + v_3)}$$

Thus the sign of $\frac{d(B/A)}{d\omega}$ depends on the elasticity of substitution of augmented capital and labor in producing the final output, together with the relative capital intensities of labor and capital-augmenting innovations. In general we have

$$\frac{d(B/A)}{d\omega} \gtrless 0 \Longleftrightarrow \frac{1}{\sigma_f - 1}\frac{\omega(v_3 - v_2)}{(\omega + v_2)(\omega + v_3)} \gtrless -1$$

Still, it is worthwhile identifying explicitly certain cases in which the "expected" result occurs in that an increase in the wage-rental ratio increases the output of labor-augmenting innovations relative to capital-augmenting innovations.

First, if both innovative activities have the same capital intensities, then B/A increases with increases in ω.

Second, $\sigma_f \geq 1$ and $v_3 \geq v_2$; that is, if labor and capital are good substitutes for one another so far as the final good is concerned, and labor-augmenting innovations are relatively more capital intensive than capital-augmenting innovations, B/A increases with ω.

Third, $\sigma_f \leq 1$ and $v_2 \geq v_3$, the case in which there is poor substitutability in producing the final good and capital-augmenting innovations are more capital intensive than labor-augmenting innovations, B/A again increases with ω.

Thus even in the special case where rates of change of innovative activity are independent of the levels achieved, the question as to whether an increase in the wage-rental ratio leads to concentration on labor-augmenting innovations rests on the empirical properties of the production relations within the firm.

Case 2. $\dot{A}$ and $\dot{B}$ independent of A and B.

A second case in which quasi-steady states occur is that in which the outputs of the innovative activities (and not simply the percentage rates of change of such outputs) are independent of the levels achieved. In terms of our model, this is the case where $\alpha(A) = C_1$, $\gamma(B) = C_2$ for some fixed constants C_1 and C_2. To simplify things, we take these constants both to be unity. By (**) we have

$$\frac{\dot{q}_1}{q_1} = \frac{\dot{A}}{A} - \frac{\dot{B}}{B} + \frac{\gamma'\dot{B}}{\gamma} - \frac{\alpha'\dot{A}}{\alpha} = \frac{\dot{A}}{A} - \frac{\dot{B}}{B},$$

on the other hand, by (*)

$$\frac{\dot{q}_1}{q_1} = (\sigma_f - 1)\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)$$

Hence if $\sigma_f \neq 2$, A/B is constant over time, with $q_1 = \frac{A}{B}\omega\left(\frac{h'}{g'}\right)$

Then
$$dq_1 = \frac{A}{B}\left(\frac{h'}{g'}\right)d\omega + \frac{h'}{g'}\omega d(A/B) + \frac{A}{B}\omega\frac{\left(g'h''dv_3 - h'g''dv_2\right)}{(g')^2}$$

with
$$\frac{1}{q_1}\frac{dq_1}{d\omega} = \frac{1}{\omega} + B/A\frac{d(A/B)}{d\omega} + \left(\frac{-1}{\omega + v_3} + \frac{1}{\omega + v_2}\right)$$

From
$$\omega = \frac{B}{A}\left(\frac{f - v_1 f'}{f'}\right)$$

$$\left(\frac{1 - \sigma_f}{\sigma_f}\right)\frac{A}{B}\frac{d(B/A)}{d\omega} = \left\{\frac{1}{\omega} - \frac{1}{\sigma_f q_1}\frac{dq_1}{d\omega}\right\}$$

Substituting for $\frac{1}{q_1}\frac{dq_1}{d\omega}$, we obtain

$$\frac{A}{B}\frac{d(B/A)}{d\omega} = -\left\{\frac{1}{\omega}\left(1-\frac{1}{\sigma_f}\right) - \frac{1}{\sigma_f}\left(\frac{1}{\omega+v_2}-\frac{1}{\omega+v_3}\right)\right\}$$

$$\omega\frac{A}{B}\frac{d(B/A)}{d\omega} = \frac{1}{\sigma_f}\left\{(1-\sigma_f)+\left(\frac{\omega}{\omega+v_2}-\frac{\omega}{\omega+v_3}\right)\right\}$$

$$= \frac{1}{\sigma_f}\left\{1-\sigma_f+\frac{\omega(v_3-v_2)}{(\omega+v_2)(\omega+v_3)}\right\}$$

Thus for $\sigma_f < 1$ and $v_3 \geq v_2$, an increase in the wage-rental ratio leads to an expansion of output of labor-augmenting innovations relative to capital-augmenting innovations, while if $\sigma_f > 1$ and $v_2 \geq v_3$ the opposite result obtains. Note that these are quite different cases than those identified in Case 1 above.

Finally, consider the case where $\alpha(A) = A^m$, $\gamma(B) = B^m$.

By (**), $\frac{\dot{q}_1}{q_1} = (1-m)\left[\frac{\dot{A}}{B}-\frac{\dot{B}}{B}\right]$.

For $\sigma_f \neq 2-m$, (*) implies $\frac{\dot{q}_1}{q_1} = 0$ with A/B constant over time,

and with $q_1 = \omega\left(\frac{h'}{g'}\right)\left(\frac{B}{A}\right)^{m-1}$

$$\text{Thus } dq_1 = \left(\frac{B}{A}\right)^{m-1}\left(\frac{h'}{g'}\right)d\omega + \frac{h'}{g'}\omega(m-1)\left(\frac{B}{A}\right)^{m-2}d(B/A) + \left(\frac{B}{A}\right)^{m-1}\omega\left[\frac{g'h''dv_3 - h'g''dv_2}{(g')^2}\right]$$

so that $\frac{1}{q_1}\frac{dq_1}{d\omega} = \frac{1}{\omega}+(m-1)\frac{A}{B}\frac{d(B/A)}{d\omega}+\left(\frac{-1}{\omega+v_3}+\frac{1}{\omega+v_2}\right)$.

From $\omega = \frac{B}{A}\left(\frac{f-v_1f'}{f'}\right)$ and with $\sigma_f \neq 1$ we have

$$\frac{A}{B}\frac{d(B/A)}{d\omega} = \frac{\sigma_f}{1-\sigma_f}\left\{\frac{1}{\omega}-\frac{1}{\sigma_f q_1}\frac{dq_1}{d\omega}\right\}.$$

Substituting for $\frac{1}{q_1}\frac{dq_1}{d\omega}$; we obtain

$$\omega\frac{A}{B}\frac{d(B/A)}{d\omega} = \frac{1}{\sigma_f - m}\left\{(1-\sigma_f)+\frac{\omega(v_3-v_2)}{(\omega+v_2)(\omega+v_3)}\right\}.$$

Clearly the cases $m = 1$ ($\alpha(A) = C_1A$, $\gamma(B) = C_2B$) and $m = 0$ ($\alpha(A) = C_1$, $\gamma(B) = C_2$) are covered by the above formula. The sign of $\frac{d(B/A)}{d\omega}$ clearly depends crucially on m.

We also note that

$$\frac{\omega}{q_1}\frac{dq_1}{d\omega} = 1+\frac{(m-1)}{\sigma_f-m}(1-\sigma_f)+\frac{\omega(v_3-v_2)}{(\omega+v_3)(\omega+v_2)}\left[\frac{\sigma_f-1}{\sigma_f-m}\right]$$

For $m = 0$,

$$\frac{\omega}{q_1}\frac{dq_1}{d\omega} = 1+\frac{\sigma_f-1}{\sigma_f}\left[1+\frac{\omega(v_3-v_2)}{(\omega+v_3)(\omega+v_2)}\right]$$

Thus for that special case at least, the response of the capital-labor ratio in producing the final product to changes in ω depends on the elasticity of substitution and on the relative capital intensities in the innovating sector.

There are no doubt other "quasi-steady state" situations where comparative dynamic results could be obtained. For example, in the Cobb-Douglas case, q_1 is determined by ω and is constant over time, but

A/B generally is time dependent in that case. What we have tried to do is to indicate the fact that in the context of a profit-maximizing model, the links between the wage-rental ratio and the direction of innovative activity are generally quite complex and that there are no obvious simple generalizations even under "quasi-steady state" conditions.

FOOTNOTES

1. Nordhaus (1969) has argued convincingly that under the assumptions we make about the manner in which innovations are produced and used, the production of innovations is only consistent with a competitive market structure under unrealistic conditions which also complicate the analysis to an unmanageable degree. On the other hand, the model we employ can be viewed as one in which an independent producer of innovations sells his innovations to a competitive industry, capturing the monopoly profits from the production of innovations. Excluding the case of "big" innovations (see Arrow (1962)), the analysis is identical in either formulation of the model.

2. Note that in this model the firm is certain about all present and future prices, and about the nature of the innovation which will result from the use of specific quantities of inputs to innovative activity.

3. Two important contributions to the theory of induced innovation by Ahmad (1966) and Nordhaus (1973) are left out of this survey because they concentrate on the classification of bias in exogenous trends.

4. We have ignored non-negativity constraints on the control and state variables because of the special assumptions to be imposed on P, F, ϕ, and Ψ as noted below.

5. The first order conditions as stated differ in two crucial respects from those obtained by maximizing the instantaneous rate of cost reduction. First, the explicit consideration of the factor inputs to innovation implies that simply considering what happens to the price of a factor used in final goods production does not provide sufficient information to allow conclusions about bias in technical progress. If that factor is also used in innovation, then additional conditions are needed. Second, the conditions for profit maximization over time depend on more than relative shares.

6. We have assumed a kind of "strong independence" with respect to the production of innovations in that ϕ is independent of B, and Ψ is independent of A. This amounts to assuming the lack of interdependence in research activities with respect to capital- and labor-augmenting innovations, and is at best a simplification that is hard to justify except in terms of the ease of manipulation of the model. The same is true of the separability assumptions relating to α and γ.

7. When prices are constant over time and $\dot{q}_1 = 0$, both factors are always augmented at the same proportional rate, independently of the size of ω. When ω is different between time paths, only the relative levels of augmentation, not the rates, are changed. This would appear to support and clarify Fellner's argument.

8. Ahmad (1966) has pointed out the contrast between the results obtained when the IPF relates $\dot{A}/A$ to $\dot{B}/B$ and the results when it relates $\dot{A}$ to $\dot{B}$, and the importance of specifying how the current innovation possibilities depend on past choices (which imply current <u>levels</u> of A and B). Nordhaus (1973) disposes of the relevance of Ahmad's conclusions to growth theory by showing that a necessary condition for balanced growth equilibrium is that the IPF $a = h(b, A, B)$ be independent of B in the sense $\partial h/\partial B = 0$. There is at least a

surface similarity between the Nordhaus condition and the cases to be examined here. However, it should be emphasized that our model is explicitly concerned with the endogenous production of innovations, while the Ahmad approach is one in which movements of the IPF are determined exogenously, subject only to their dependence on the levels of A and B already achieved. It might be that there is some closer connection between Nordhaus's notion of independence and that employed here, but it has not been possible as yet to discover such a connection.

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